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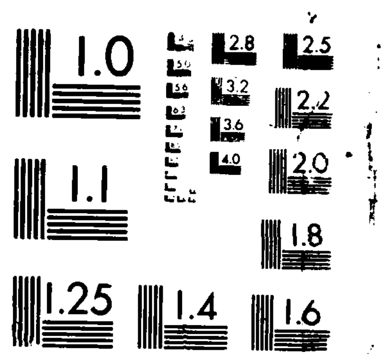

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AN APPLICATION OF FUZZY SET THEORY TO STATISTICAL HYPOTHESIS TESTING

WLLIAM E. BAKER

JUNE 1987

US ARMY BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

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In many instances the data used in statistical hypothesis testing may be vague or imprecise and, as such, may suggest results that are incorrect. Rank tests, in particular, seem susceptible, since the original data, once ranked, have no further influence on the testing procedure no matter how closely they are grouped. A possible solution is to treat the ranks as fuzzy integers represented by membership functions that indicate the degree to which each rank assumes each integer value. In this paper, a method is suggested for obtaining these membership functions; and the concept is incorporated into an existing rank test. An application of this fuzzy hypothesis-testing procedure is provided.

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I. INTRODUCTION

Suppose we have the following set of data:

$$\{-0.888, 0.200, -1.000, -0.417, -0.052, 0.186, 0.067, -0.467, -0.623, -0.181\} . \quad (1)$$

By considering their absolute values, we obtain a set S consisting of ordered pairs,

$$S = \{(1, -0.052), (2, 0.067), (3, -0.181), (4, 0.186), (5, 0.200), (6, -0.417), \quad (2)$$
$$(7, -0.467), (8, -0.623), (9, -0.888), (10, -1.000)\} ,$$

where the first member of each ordered pair is the ranking (smallest to largest) of the absolute value of the second member of the ordered pair. This type of data is often used in rank tests, nonparametric hypothesis tests which generally examine the mean or median of a distribution or the equality of means or medians of several distributions. Rank tests are sometimes eschewed because once the ranking has been established, the data are treated as though they were equally spaced; and potentially-valuable information concerning the proximity of the data points is discarded. In the preceding example, note that some of the rankings may be tenuous; for example, ranks 3 and 4 could easily have been permuted had the numbers to which they correspond been inaccurate in the third decimal place. Therefore, the degree of accuracy in the ranks is directly related to the degree of accuracy of the original data; and this can sometimes be a problem.

In many applications, the available data may be vague or imprecise, due to a variety of reasons which may include improper calibration of equipment and subjectivity of the experimenter. This, of course, can lead to imprecise ranking of the data and possibly an incorrect conclusion from the resulting hypothesis test. Such data, as well as their ranks, can be represented by fuzzy numbers[1] - a relatively new concept in which a number is described by a central value along with a spread about that value. When applied to ranks, this technique may overcome the previously-mentioned problem inherent in rank tests; and in certain situations this representation will allow for a more realistic approach to hypothesis testing.

II. FUZZY RANKS APPLIED TO THE WILCOXON SIGNED-RANKS TEST

A. Wilcoxon Signed-Ranks Test

The Wilcoxon signed-ranks test is a nonparametric hypothesis test which is generally used to test for equal medians of two distributions. The data consist of paired observations (x_i, y_i) from the two distributions. The differences between the observations, $D_i = x_i - y_i$, are then calculated; and their absolute values are assigned a rank R_i from smallest to largest. Finally, R_i is multiplied by -1 if D_i is negative. The sum of the ranks of the positive differences, $T = \sum R_i$, $R_i > 0$, is the test statistic. If the two distributions have the same median, we would expect about one-half of the D_i's

[1] Zadeh, L.A., "Fuzzy Sets," Information and Control, Vol 8, 1965

to be positive. Very high or very low values of T indicate that numbers from the first distribution are consistently higher or consistently lower than those from the second distribution and, therefore, will cause rejection of the null hypothesis of equal medians. The theory behind the test along with tables containing various quantiles of T are provided by Conover[2].

For each ordered pair of the set S, we can consider the second value to be D_i and the first value to be the R_i associated with it. Taking the sum of the R_i's associated with the positive D_i's, we find that $T = 2+4+5 = 11$. Probability levels for the Wilcoxon signed-ranks test for a sample of size 10 are given in Table 1. Referring to this table, we find that our value of T indicates that there is insufficient evidence for rejecting the hypothesis of equal medians at a 10% level of significance. In this case the probability of T being less than or equal to 11 is 0.0527; and since we are performing a two-sided test (examining T to see if its value is either too low or too high), we double that figure to get the critical level of the test. Had the value of T been 10 or less, rejection of the null hypothesis would have been warranted.

TABLE 1. Probability Levels for the Wilcoxon Signed-Ranks Test Statistic with a Sample Size of 10. *

T	P	T	P	T	P	T	P
0	.0010	7	.0186	14	.0967	21	.2783
1	.0020	8	.0244	15	.1162	22	.3125
2	.0029	9	.0322	16	.1377	23	.3477
3	.0049	10	.0420	17	.1611	24	.3848
4	.0068	11	.0527	18	.1875	25	.4229
5	.0098	12	.0654	19	.2158	26	.4609
6	.0137	13	.0801	20	.2461	27	.5000

T = sum of positive ranks

P = probability that the sum of positive ranks will be less than or equal to T under the null hypothesis

* Since the distribution of T is symmetrical, only one-half of the distribution is tabulated.

B. Fuzzy Ranks

Fuzzy set theory was introduced by Zadeh[1] over twenty years ago. In this application we will examine fuzzy numbers and, in particular, fuzzy integers since we are concerned with ranks. A fuzzy number will be represented by a membership function quantifying the degree to which it takes on any specific value. Figure 1 shows

[2] Conover, W. J., Practical Nonparametric Statistics, John Wiley and Sons, Inc., 1971.

a membership function μ for "fuzzy six". This function assumes its maximum value at six, $\mu(6) = 1$; the closer any number is to six, the higher its degree of membership in "fuzzy six". When we examine fuzzy ranks, the membership functions will be discrete, since our interest will be only in the degree of membership for integer values.

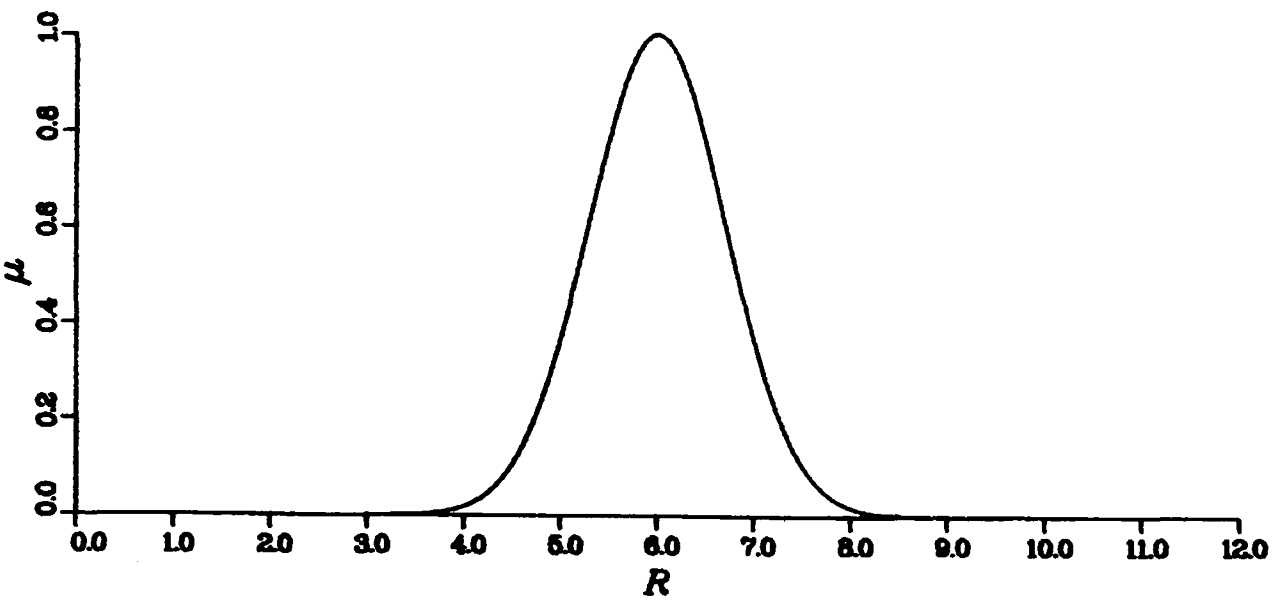

Figure 1. Membership Function of Fuzzy Six.

This membership function is not unique; rather, it is subjective - determined by the user and based on his perception of the vagueness of the data. In order to fully utilize this methodology, the Extension Principle[3] permits definition of a mathematical operation f on two fuzzy numbers. It states that if X is a fuzzy number with membership function $\mu_X(x)$ and Y is a fuzzy number with membership function $\mu_Y(y)$, then Z = f (X,Y) is a fuzzy number with membership function

$$\mu_Z(z) = \max_{\substack{x,y \\ f(x,y)=z}} \min\left[\mu_X(x), \mu_Y(y)\right] . \tag{3}$$

[3] Zadeh, L.A., "The Concept of a Linguistic Variable and its Application to Approximate Reasoning I, II, III," Information Sciences, Vols 8, 9, 1975

Figure 2 shows some membership functions established for the absolute value of three of the members of the original data set (-0.181, 0.186, 0.200). Recall that the set S contained ordered pairs of the form (I_X,X) where X was a number from the original data set and I_X was the rank associated with the absolute value of X. The shapes of these membership functions are symmetric and triangular with a spread equal to ten percent of the largest value in the data set (remember that these are modeling decisions). Hence, the membership value of "fuzzy 0.181" is non-zero from 0.081 to 0.281 and has its zenith at 0.181.

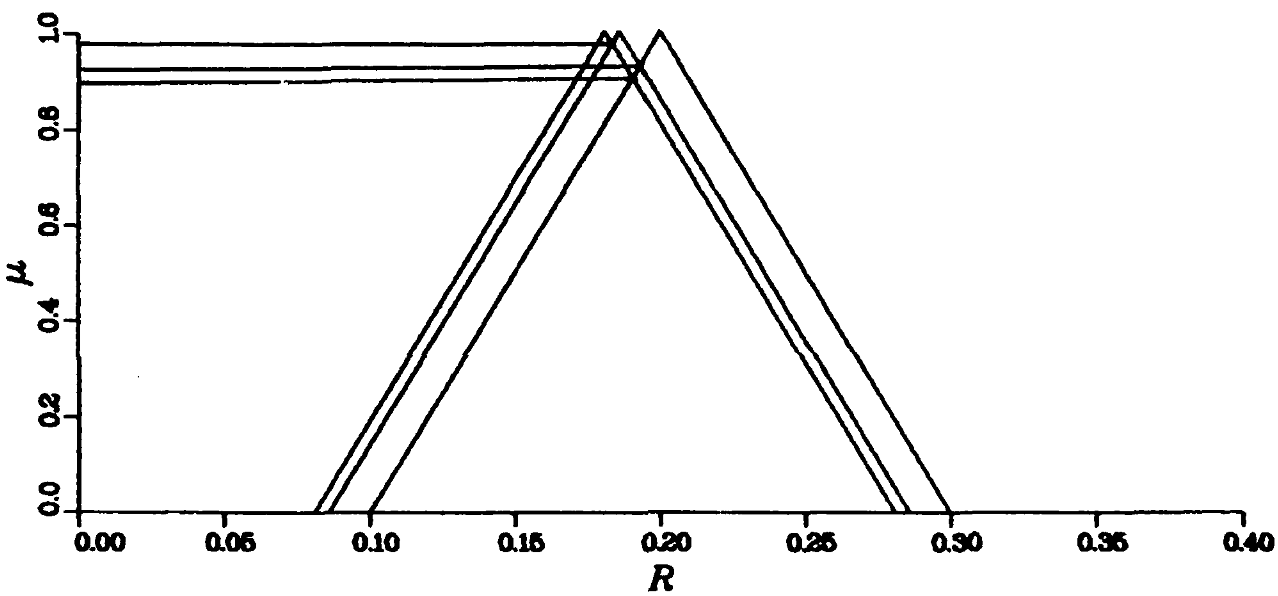

Figure 2. Membership Functions of a Portion of the Original Data Set.

We can define a membership function for the first member of each ordered pair - the rank denoted by I_X - as follows:

$$\mu_{I_X}(I_Y) = \max_{z \in R} \min\left[\mu_X(z), \mu_Y(z)\right] . \tag{4}$$

This equation provides the membership value for I_Y in "fuzzy rank I_X". Thus, in Figure 2, the top horizontal line intersects the ordinate at a point equal to $\mu_3(4)$, the middle horizontal line intersects the ordinate at a point equal to $\mu_4(5)$, and the bottom horizontal line intersects the ordinate at a point equal to $\mu_3(5)$. This definition of the membership function for the fuzzy ranks produces the following properties:

$$\mu_{I_X}(I_X) = 1, \tag{5}$$

$$\mu_{I_X}(I_Y) = 0 \text{ if } \mu_X(x) \text{ and } \mu_Y(y) \text{ do not intersect, and} \tag{6}$$

$$\mu_{I_X}(I_Y) = \mu_{I_Y}(I_X)\,. \tag{7}$$

Figure 3 shows the membership functions for the entire set of original data. The ordinate values of their points of intersection are listed in Table 2. These, of course, are the values of $\mu_{I_X}(I_Y)$ shown in Equation 4 and define the membership functions of the fuzzy ranks of the data, such functions being discrete since the ranks can take on only integer values. Note that the table is symmetric, a result of Equation 7.

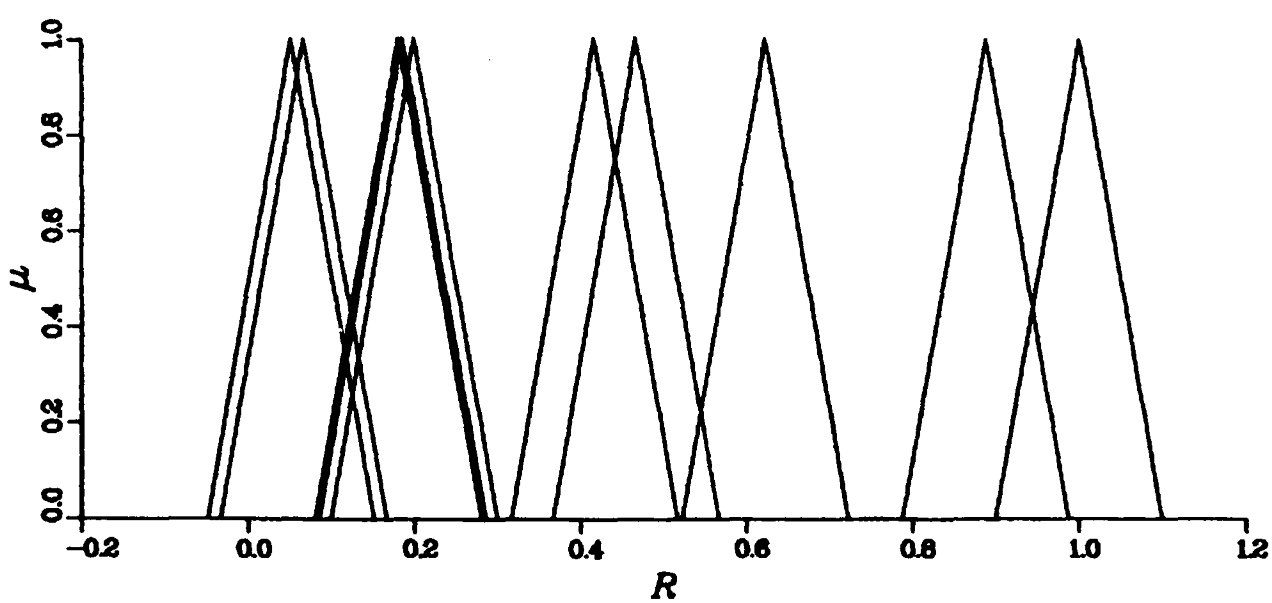

Figure 3. Membership Functions of the Original Data Set.

TABLE 2. Membership Functions Associated with the Fuzzy Ranks for the Original Data Set.

Ranked Data Points	1	2	3	4	5	6	7	8	9	10
1	1.00	0.93	0.36	0.33	0.26	0.00	0.00	0.00	0.00	0.00
2	0.93	1.00	0.43	0.41	0.34	0.00	0.00	0.00	0.00	0.00
3	0.36	0.43	1.00	0.98	0.91	0.00	0.00	0.00	0.00	0.00
4	0.33	0.41	0.98	1.00	0.93	0.00	0.00	0.00	0.00	0.00
5	0.26	0.34	0.91	0.93	1.00	0.00	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.00	1.00	0.75	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.75	1.00	0.22	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.22	1.00	0.00	0.00
9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.44
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.44	1.00

C. Incorporating Fuzzy Ranks into the Wilcoxon Signed-Ranks Test

Once the membership functions of the ranks are established, it is necessary to calculate the value of T, the sum of the positive ranks. T will be the sum of fuzzy integers and, as such, will be a fuzzy integer itself. To determine its membership function, we refer to the Extension Principle and determine that

$$\mu_T(t) = \max_{\substack{(I_{Y_1}, I_{Y_2}, \ldots, I_{Y_{10}}) \\ t = \sum_{i=1}^{10} I_{Y_i}\,,\ Y_i > 0}} \min\,[\mu_1(I_{Y_1}), \mu_2(I_{Y_2}), \ldots, \mu_{10}(I_{Y_{10}})]\,, \tag{8}$$

where $(I_{Y_1}, I_{Y_2}, \ldots, I_{Y_{10}})$ denotes all permutations of the integers $I_{Y_1}, I_{Y_2}, \ldots, I_{Y_{10}}$.

In this case of ten data points, T can take on all integer values between 0 and 55; each of these possible sums will have a membership value associated with it. To obtain $\mu_T(t)$, we refer to Table 2 and perform the following steps:

1. Select a permutation of the ranks.

2. From Table 2 determine the minimum membership value of the ranks in their respective positions for this particular permutation.

3. If that minimum membership value is greater than zero, determine the sum of the positions of the positive ranks for this particular permutation.

4. If the membership value is greater than the membership value currently associated with that sum, replace with the new membership value.

We continue with this sequence of operations until all the permutations have been exhausted, at which time we have associated with every possible value of T a membership value which is the maximum over all permutations of the minimums for each individual permutation.

Using our set of ordered pairs, S, we can provide an example of the sequence above:

1. Suppose our selected permutation is 5 1 3 2 4 7 6 8 10 9.

2. Referring to Table 2, we can see that the membership value of rank 5 in the first position is 0.26, the membership value of rank 1 in the second position is 0.93, the membership value of rank 3 in the third position is 1.00, and so forth. If any one of these is equal to zero, then the minimum is equal to zero, and we skip steps three and four. For this particular permutation, the minimum membership value is 0.26.

3. The sum of the positions of the positive ranks for this particular permutation is equal to ten (first plus fourth plus fifth).

4. If 0.26 is greater than the current membership value associated with a sum of ten, then replace it.

When we have examined all possible permutations, the membership function associated with the sum of positive ranks, T, is shown in Table 3. Membership values associated with $T \leq 5$ and $T \geq 13$ are all equal to zero.

TABLE 3. Membership Function Associated with the Sum of Positive Ranks for the Original Data Set (Non-zero Values).

T	$\mu(T)$
6	0.330
7	0.355
8	0.905
9	0.925
10	0.975
11	1.000
12	0.430

Of course, examining all permutations can be very time consuming. This particular case required 201 seconds of central processor unit (CPU) time on a CDC 7600 computer. However, because of the large number of membership values that were equal to zero (see Table 2), many of the permutations could be ignored, since resulting minimums would be equal to zero and would not affect subsequent maximums. By taking advantage of this information to modify the permutation subroutine, I was able to reduce the CPU requirement to 43 seconds. Even with this kind of reduction, it is difficult to exceed a sample size of twelve without incorporating other shortcuts. One very effective method is to segment the data set, particularly if there is a datum point which is crisp rather than fuzzy; that is, its membership value at all but one position is equal to zero. Using this characteristic, I was able to handle a sample size of 32 in a succeeding section dealing with an application of this work.

III. INTERPRETING RESULTS

When the data were considered non-fuzzy, we saw that there was insufficient evidence for rejecting the hypothesis of equal medians. We could have provided a critical level as defined by Conover; in doing so, we would have concluded that the null hypothesis could have been rejected at a significance level of 10.54% (see Table 1 and recall that we are performing a two-sided test).

Treating the data as fuzzy numbers provides a fuzzy result for T with a membership function described in Table 3. This allows for several methods of interpretation. Observing that $\mu(T) = 1$ (its maximum) when T=11, we might state that there is insufficient evidence for rejecting the null hypothesis at the $\alpha = .10$ level. Thus, the classical (non-fuzzy) signed-ranks test emerges as a special case. Alternatively, knowing that T=10 was the threshold for rejection, we might state that the null hypothesis can be rejected at the $\alpha = .10$ level with a membership value of 0.975. Since we recognize the data as imprecise, perhaps the best alternative is to accept the imprecision inherent in the resulting test statistic and make the decision as to whether or not to reject the null hypothesis based on the entire membership function. In our example, the membership value exceeds 0.900 for T=8 through T=11. Therefore, none of these values should be disregarded when analyzing the data; they all became viable candidates for T when the model took into account the proximity of the data points. The nature of any particular application should assist in making the final decision less subjective. Our example represents a situation in which the null hypothesis of equal medians would not have been rejected based on the original data set but may be rejected when the data, imprecise in nature, are treated as fuzzy numbers.

IV. APPLICATION

In a report on statistical methods of computer simulation validation[4], the Wilcoxon

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[4] Baker, W.E. and Taylor, M.S., A Nonparametric Statistical Approach to the Validation of Computer Simulation Models, BRL-TR-2696, November 1985, AD#A163376

signed-ranks test was used to examine paired observations; in this case, empirical data versus simulation results. The null hypothesis of equal medians was loosely stated as "the values of the empirical data tend to agree with the values of the simulation results" which we can interpret as "the simulation model is valid." Results showed that there was insufficient evidence to reject the null hypothesis at the $\alpha = .10$ level for the two-sided test. In fact, the critical value was 0.246 meaning that rejection of the null hypothesis based on this data set would provide only a 24.6% level of significance. The data and results are shown in Table 4. Note that there are 32 values including eight zeros and one tie. A method for incorporating these phenomena into the test is provided by Lehmann[5]. Primarily, this method consists of the zeros assuming a rank of zero and the tied values assuming a rank equal to the average of the ranks which would normally have been assigned to them.

These data, consisting of probabilities of kill against a target vehicle, have a tendency to be vague and imprecise. This can be due to the subjectivity of the vulnerability analyst who provides the empirical results through his estimates of damage, the inability to model all of the relevant input factors in the computer simulation, and other, more subtle reasons. Treating the differences in Table 4 as fuzzy numbers, I decided on a triangular-shaped membership function with a spread of 0.02 on each side of the central value. The intersections of all membership functions for this set of data are presented in Table 5. Recall that this matrix of values defines the discrete membership functions for the fuzzy ranks; for example, the fuzzy number associated with rank 9 takes on rank 10 with a membership value of 0.93. This implies that fuzzy rank 9 assumes position number 10 with a membership value of 0.93.

Thus, an element of Table 5, which can be designated as $\mu_{I_X}(I_Y)$, is equal to the membership value for I_Y in fuzzy rank I_X. Using this matrix, the value of $T = \sum R_i$ can be calculated. Because the number of permutations is enormous, the data set must be segmented. A reasonable place to separate is between the differences in Table 4 of -0.067 and -0.107. The former is associated with rank 19, and the latter is associated with rank 20. When these ranks are considered fuzzy as in Table 5, then $\mu_{19}(20) = \mu_{20}(19) = 0$. Also, we can isolate the differences 0.165 (rank 23), 0.487 (rank 27), 0.619 (rank 30), -0.736 (rank 31), and -0.950 (rank 32), since in each case $\mu_i(j) = 0$ for all $j \neq i$. As in the earlier example, fuzzy arithmetic then provides the membership function for T; this is reproduced in Table 6.

Note that $\mu(T) = 1$ when T=327; again, this is merely the classical result. Although the evidence remains insufficient to reject at the $\alpha = .10$ level, we could reject at the $\alpha = .20$ level with a membership value of 0.378. However, the high membership values (greater than 0.900) occur for T=324 through T=331; and so it would be careless to reject the null hypothesis even at the 20% significance level after considering the entire membership function.

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[5] Lehmann, E.L., Nonparametrics: Statistical Methods Based on Ranks, Holden-Day, Inc. 1975

TABLE 4. Wilcoxon Signed-Ranks Test Applied to the Validation of a Computer Simulation Model.

Shot Number	Empirical Value	Simulation Result	Difference	Signed Rank of Difference
43	0.734	0.719	0.015	11
44	0.145	0.700	-0.555	-29
45	1.000	1.000	0.000	0
46	1.000	1.000	0.000	0
47	0.100	0.116	-0.016	-12
48	0.900	0.776	0.124	22
49	0.930	0.655	0.275	25
50	1.000	1.000	0.000	0
51	0.145	0.881	-0.736	-31
52	1.000	0.967	0.033	16
53	0.668	0.503	0.165	23
54	1.000	1.000	0.000	0
55	1.000	0.890	0.110	21
56	0.905	0.286	0.619	30
57	0.550	0.523	0.027	14.5
58	1.000	0.986	0.014	10
59	1.000	0.457	0.543	28
60	0.050	1.000	-0.950	-32
62	1.000	0.973	0.027	14.5
64	0.100	0.207	-0.107	-20
65	1.000	1.000	0.000	0
66	0.668	0.735	-0.067	-19
67	0.953	0.970	-0.017	-13
68	1.000	0.738	0.262	24
69	1.000	1.000	0.000	0
70	1.000	0.949	0.051	18
71	1.000	0.513	0.487	27
72	1.000	0.958	0.042	17
73	1.000	1.000	0.000	0
74	0.905	0.608	0.297	26
75	0.668	0.679	-0.011	-9
76	1.000	1.000	0.000	0

$\sum$ Positive Ranks = 327

Critical T-values ($\alpha = 0.05 \times 2$) = 142 (lower), 350 (upper)

Critical T-values ($\alpha = 0.10 \times 2$) = 158 (lower), 334 (upper)

TABLE 5. Membership Functions Associated with the Fuzzy Ranks for the Differences in Probabilities of Kill.

Ranked Data Points	9	10	11	12	13	14.5	14.5	16	17	18	19	20
9	1.00	0.93	0.90	0.88	0.85	0.60	0.60	0.45	0.23	0.00	0.00	0.00
10	0.93	1.00	0.98	0.95	0.93	0.68	0.68	0.53	0.30	0.08	0.00	0.00
11	0.90	0.98	1.00	0.98	0.95	0.70	0.70	0.55	0.33	0.10	0.00	0.00
12	0.88	0.95	0.98	1.00	0.98	0.73	0.73	0.58	0.35	0.13	0.00	0.00
13	0.85	0.93	0.95	0.98	1.00	0.75	0.75	0.60	0.38	0.15	0.00	0.00
14.5	0.60	0.68	0.70	0.73	0.75	1.00	1.00	0.85	0.63	0.40	0.00	0.00
14.5	0.60	0.68	0.70	0.73	0.75	1.00	1.00	0.85	0.63	0.40	0.00	0.00
16	0.45	0.53	0.55	0.58	0.60	0.85	0.85	1.00	0.78	0.55	0.15	0.00
17	0.23	0.30	0.33	0.35	0.38	0.63	0.63	0.78	1.00	0.78	0.38	0.00
18	0.00	0.08	0.10	0.13	0.15	0.40	0.40	0.55	0.73	1.00	0.60	0.00
19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.38	0.60	1.00	0.00
20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.92
22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.57
23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
24	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
27	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
28	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
29	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
31	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
32	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

First eight values are zero
Includes one tie

TABLE 5. Membership Functions Associated with the Fuzzy Ranks for the Differences in Probabilities of Kill (Continued).

Ranked Data Points	21	22	23	24	25	26	27	28	29	30	31	32
9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
14.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
14.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	0.92	0.57	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
21	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
22	0.65	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
23	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
24	0.00	0.00	0.00	1.00	0.67	0.12	0.00	0.00	0.00	0.00	0.00	0.00
25	0.00	0.00	0.00	0.67	1.00	0.45	0.00	0.00	0.00	0.00	0.00	0.00
26	0.00	0.00	0.00	0.12	0.45	1.00	0.00	0.00	0.00	0.00	0.00	0.00
27	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
28	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.70	0.00	0.00	0.00
29	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.70	1.00	0.00	0.00	0.00
30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
31	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
32	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

First eight values at zero
Includes one tie

TABLE 6. Membership Function Associated with the Sum of Positive Ranks of the Differences in Probabilities of Kill (Non-zero Values).

T	μ(T)	T	μ(T)	T	μ(T)
308	0.154	318	0.702	328	0.975
309	0.154	319	0.702	329	0.950
310	0.154	320	0.726	330	0.950
311	0.378	321	0.726	331	0.950
312	0.378	322	0.751	332	0.700
313	0.378	323	0.751	333	0.602
314	0.575	324	0.901	334	0.378
315	0.602	325	0.925	335	0.378
316	0.602	326	0.925	336	0.154
317	0.602	327	1.000	337	0.005

V. SUMMARY

Hypothesis testing is an important and useful tool for data analysis. When the data are vague or imprecise, an additional source of error is introduced and may result in an incorrect decision whether or not to reject the null hypothesis. Treating the data as fuzzy numbers allows us to model the uncertainty; and manipulating the data using fuzzy arithmetic allows us to carry the uncertainty through to the final results, at which point a more informed decision can be made.

Rank Tests are a class of hypothesis tests which are especially susceptible to the problems of imprecise data since the data, once ranked, have no further influence regardless of how closely they might be grouped. The Wilcoxon signed-ranks test is one example; and it was this particular hypothesis test that was applied to some admittedly-imprecise vulnerability data. The data were represented as fuzzy numbers, and the test statistic was calculated using fuzzy arithmetic. This provided a final result which was itself a fuzzy number, and several methods of interpreting this result were discussed.

I found computer time to be a major problem with incorporating fuzzy data into rank tests. In this case I needed to examine all possible permutations of rankings for all the data. For 10 data points the problem is not too bad; but if the data set is expanded to 32 points (as with the vulnerability data), then even with newer, faster computers some special techniques must be applied. In most cases one should be able to segment the data set, so that groups of ten or less can be examined and the results combined. This should make fuzzy hypothesis testing feasible as well as reasonable -- an even more important and more useful tool for the statistician!

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